GCSE subject criteria for mathematics

March 2009

Ofqual/08/3997

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Introduction

1. GCSE subject criteria set out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in mathematics. They provide the framework within which an awarding body creates the detail of the specification.

2. Specifications must also meet the regulators' general requirements, including the common and GCSE criteria as defined in *The statutory regulation of external qualifications* (QCA/04/1293).

3. Subject criteria are intended to:

   - help ensure consistent and comparable standards in the same subject across the awarding bodies
   - ensure that the rigour of GCSE is maintained
   - ensure that specifications build on the knowledge, understanding and skills established by the national curricula for England, Northern Ireland and Wales, and facilitate progression to higher level qualifications in mathematics
   - help higher education institutions, employers and other stakeholders, such as learners and parents/guardians, know what has been studied and assessed.

4. Any GCSE specification that contains significant elements of mathematics must be consistent with the relevant parts of these subject criteria.

Aims and learning outcomes

5. GCSE specifications in mathematics should encourage learners to be inspired, moved and changed by following a broad, coherent, satisfying and worthwhile course of study. They should help learners to develop confidence in, and a positive attitude towards, mathematics and to recognise the importance of mathematics in their own lives and to society. Specifications should prepare learners to make informed decisions about the use of technology, the management of money, further learning opportunities and career choices.

6. GCSE specifications in mathematics must enable learners to:

   - develop knowledge, skills and understanding of mathematical methods and concepts
   - acquire and use problem-solving strategies

- select and apply mathematical techniques and methods in mathematical, everyday and real-world situations
- reason mathematically, make deductions and inferences and draw conclusions
- interpret and communicate mathematical information in a variety of forms appropriate to the information and context.

Subject content

7. The content of GCSE specifications in mathematics must reflect the learning outcomes.

8. GCSE specifications in mathematics must be consistent with the national curriculum key stage 4 programmes of study requirements in the orders for England and Wales, and the statutory requirements for key stage 4 in Northern Ireland.

9. GCSE specifications in mathematics must enable learners to develop the knowledge, skills and understanding specified below. **Higher tier content is in bold type.**

Number and algebra

- add, subtract, multiply and divide any number
- order rational numbers
- use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition
- use the terms square, positive and negative square root, cube and cube root
- use index notation for squares, cubes and powers of 10
- use index laws for multiplication and division of integer, **fractional and negative** powers
- **interpret, order and calculate with numbers written in standard index form**
- understand equivalent fractions, simplifying a fraction by cancelling all common factors
- add and subtract fractions
- use decimal notation and recognise that each terminating decimal is a fraction

- recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals
- understand that ‘percentage’ means ‘number of parts per 100’ and use this to compare proportions
- use percentage, **repeated proportional change**
- **understand and use direct and indirect proportion**
- interpret fractions, decimals and percentages as operators
- use ratio notation, including reduction to its simplest form and its various links to fraction notation
- understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations
- **use surds and π in exact calculations**
- **calculate upper and lower bounds**
- divide a quantity in a given ratio
- approximate to specified or appropriate degrees of accuracy including a given power of ten, number of decimal places and significant figures
- use calculators effectively and efficiently, including statistical **and trigonometrical** functions
- distinguish the different roles played by letter symbols in algebra, using the correct notation
- distinguish in meaning between the words equation, formula, **identity** and expression
- manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, and by taking out common factors, **multiplying two linear expressions, factorising quadratic expressions including the difference of two squares, and simplifying rational expressions**
- set up and solve simple equations **including simultaneous equations in two unknowns**
- **solve quadratic equations**
- derive a formula, substitute numbers into a formula and change the subject of a formula
- solve linear inequalities in one **or two** variables, and represent the solution set on a number line **or suitable diagram**

- use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them
- generate terms of a sequence using term-to-term and position-to-term definitions of the sequence
- use linear expressions to describe the *n*th term of an arithmetic sequence
- use the conventions for coordinates in the plane and plot points in all four quadrants, including using geometric information
- recognise and plot equations that correspond to straight-line graphs in the coordinate plane, including finding gradients
- **understand that the form *y* = *mx* + *c* represents a straight line and that *m* is the gradient of the line and *c* is the value of the *y*-intercept**
- **understand the gradients of parallel lines**
- **find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions**
- **draw, sketch, recognise graphs of simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the function $y = k^x$ for integer values of *x* and simple positive values of *k*, the trigonometric functions *y* = sin *x* and *y* = cos *x***
- **construct the graphs of simple loci**
- construct linear**, quadratic and other** functions from real-life problems and plot their corresponding graphs
- discuss, plot and interpret graphs (which may be non-linear) modelling real situations
- generate points and plot graphs of simple quadratic functions, and use these to find approximate solutions.

Geometry and measures

- recall and use properties of angles at a point, angles at a point on a straight line (including right angles), perpendicular lines, and opposite angles at a vertex
- understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals

- calculate and use the sums of the interior and exterior angles of polygons
- recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus
- recognise reflection and rotation symmetry of 2D shapes
- understand congruence and similarity
- use Pythagoras' theorem in 2D **and 3D**
- **use the trigonometrical ratios and the sine and cosine rules to solve 2D and 3D problems**
- distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- **understand and construct geometrical proofs using circle theorems**
- use 2D representations of 3D shapes
- describe and transform 2D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor **then use positive fractional and negative scale factors** and distinguish properties that are preserved under particular transformations
- use and interpret maps and scale drawings
- understand **and use** the effect of enlargement for perimeter, area and volume of shapes and solids
- interpret scales on a range of measuring instruments and recognise the inaccuracy of measurements
- convert measurements from one unit to another
- make sensible estimates of a range of measures
- understand and use bearings
- understand and use compound measures
- measure and draw lines and angles
- draw triangles and other 2D shapes using a ruler and protractor
- use straight edge and a pair of compasses to do constructions
- construct loci

- calculate perimeters and areas of shapes made from triangles and rectangles **and other shapes**
- **calculate the area of a triangle using $\frac{1}{2}ab$ sin C**
- find circumferences and areas of circles
- calculate volumes of right prisms and of shapes made from cubes and cuboids
- **solve mensuration problems involving more complex shapes and solids.**

Statistics and probability

- understand and use statistical problem solving process/handling data cycle
- identify possible sources of bias
- design an experiment or survey
- design data-collection sheets distinguishing between different types of data
- extract data from printed tables and lists
- design and use two-way tables for discrete and grouped data
- produce charts and diagrams for various data types
- calculate median, mean, range, **quartiles and inter-quartile range**, mode and modal class
- interpret a wide range of graphs and diagrams and draw conclusions
- look at data to find patterns and exceptions
- recognise correlation and draw and/or use lines of best fit by eye, understanding what these represent
- compare distributions and make inferences
- understand and use the vocabulary of probability and the probability scale
- understand and use estimates or measures of probability from theoretical models (including equally likely outcomes), or from relative frequency
- list all outcomes for single events, and for two successive events, in a systematic way and derive related probabilities
- identify different mutually exclusive outcomes and know that the sum of the probabilities of all these outcomes is 1

- **know when to add or multiply two probabilities: if *A* and *B* are mutually exclusive, then the probability of *A* or *B* occurring is P(*A*) + P(*B*), whereas if *A* and *B* are independent events, the probability of *A* and *B* occurring is P(*A*) × P(*B*)**
- **use tree diagrams to represent outcomes of compound events, recognising when events are independent**
- compare experimental data and theoretical probabilities
- understand that if they repeat an experiment, they may — and usually will — get different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics

Assessment objectives

10. All specifications in mathematics must require learners to demonstrate their ability to:

Assessment objectives		Weighting (%)
AO1	■ recall and use their knowledge of the prescribed content	45–55
AO2	■ select and apply mathematical methods in a range of contexts	25–35
AO3	■ interpret and analyse problems and generate strategies to solve them.	15–25

Scheme of assessment

11. GCSE specifications in mathematics must allocate a weighting of 100 per cent to external assessment.

12. Question papers in mathematics must be targeted at either foundation or higher tier.

13. Each scheme of assessment must allocate a minimum weighting of 25 per cent, and a maximum weighting of 50 per cent, to assessment without a calculator.

14. GCSE assessments in mathematics must allocate a weighting of 20-30 per cent on higher tier and 30-40 per cent on foundation tier for the functional elements of mathematics.

Grade descriptions

15. Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content in the specification; they are not designed to define that content.

16. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of candidates' performance in the assessment may be balanced by better performances in others.

Grade	Description
A	Candidates use a wide range of mathematical techniques, terminology, diagrams and symbols consistently, appropriately and accurately. Candidates are able to use different representations effectively and they recognise equivalent representations for example numerical, graphical and algebraic representations. Their numerical skills are sound, they use a calculator effectively and they demonstrate algebraic fluency. They use trigonometry and geometrical properties to solve problems. Candidates identify and use mathematics accurately in a range of contexts. They evaluate the appropriateness, effectiveness and efficiency of different approaches. Candidates choose methods of mathematical communication appropriate to the context. They are able to state the limitations of an approach or the accuracy of results. They use this information to inform conclusions within a mathematical or statistical problem. Candidates make and test hypotheses and conjectures. They adopt appropriate strategies to tackle problems (including those that are novel or unfamiliar), adjusting their approach when necessary. They tackle problems that bring together different aspects of mathematics and may involve multiple variables. They can identify some variables and investigate them systematically; the outcomes of which are used in solving the problem.

	Candidates communicate their chosen strategy. They can construct a rigorous argument, making inferences and drawing conclusions. They produce simple proofs and can identify errors in reasoning.
C	Candidates use a range of mathematical techniques, terminology, diagrams and symbols consistently, appropriately and accurately. Candidates are able to use different representations effectively and they recognise some equivalent representations eg numerical, graphical and algebraic representations of linear functions; percentages, fractions and decimals. Their numerical skills are sound and they use a calculator accurately. They apply ideas of proportionality to numerical problems and use geometric properties of angles, lines and shapes. Candidates identify relevant information, select appropriate representations and apply appropriate methods and knowledge. They are able to move from one representation to another, in order to make sense of a situation. Candidates use different methods of mathematical communication. Candidates tackle problems that bring aspects of mathematics together. They identify evidence that supports or refutes conjectures and hypotheses. They understand the limitations of evidence and sampling, and the difference between a mathematical argument and conclusions based on experimental evidence. They identify strategies to solve problems involving a limited number of variables. They communicate their chosen strategy, making changes as necessary. They construct a mathematical argument and identify inconsistencies in a given argument or exceptions to a generalisation.
F	Candidates use some mathematical techniques, terminology, diagrams and symbols from the foundation tier consistently, appropriately and accurately. Candidates use some different representations effectively and can select information from them. They complete straightforward calculations competently with and without a calculator. They use simple fractions and percentages, simple formulae and some geometric properties, including symmetry. Candidates work mathematically in everyday and meaningful contexts. They make use of diagrams and symbols to communicate mathematical ideas. Sometimes, they check the accuracy and

	reasonableness of their results. Candidates test simple hypotheses and conjectures based on evidence. Candidates are able to use data to look for patterns and relationships. They state a generalisation arising from a set of results and identify counter-examples. They solve simple problems, some of which are non-routine.